Letters

Comments on "Some Pitfalls in Millimeter-Wave Noise Measurements Utilizing a Cross-Correlation Receiver"

H. J. SIWERIS, B. SCHIEK, AND K. M. LÜDEKE

In the above paper,[1] Sutherland and van der Ziel analyze some problems encountered during noise measurements at very low temperatures with a cross-correlation receiver. It is shown that, if a hybrid junction is used to split the noise signal from the device under test (DUT) into the two receiver channels, the correlation between the receiver input signals, which contains the useful information, vanishes if the remaining port of the hybrid is resistively terminated at a temperature equal to that of the DUT. The same effect occurs for a pure reactive termination, if the isolators used to decouple the receiver channels are also cooled down to the temperature of the DUT.

These results are absolutely correct but, as will be pointed out in this comment, they could have been obtained in a more straightforward and general way by the application of a very useful theorem, which was published by Bosma [1] already in 1967, but which does not seem to have received the appreciation it deserves. This theorem relates the Nyquist noise waves emanating from the ports of a passive, linear, but not necessarily reciprocal n-port at a homogeneous temperature T, to the scattering matrix $[S]$ of the network. Explicitly, for the noise waves X_i and X_j at ports i and j, respectively, it states that

$$\langle X_i X_j^* \rangle = kT\Delta f N_{ij} \tag{1}$$

with N_{ij} denoting the elements of the noise-distribution matrix

$$[N] = [I] - [S][S]^x \tag{2}$$

where $[I]$ is the identity matrix and $[S]^x$ the complex conjugate of the transposed scattering matrix. Thus, the cross-correlation of the noise waves X_1 and X_2 of a two-port is given by

$$\langle X_1 X_2^* \rangle = -kT\Delta f(S_{11}S_{21}^* + S_{12}S_{22}^*). \tag{3}$$

It follows from (3) that for all linear passive two-ports at a uniform temperature, the correlation vanishes if $S_{11} = S_{22} = 0$, i.e., if the two-port is perfectly matched. The same holds if the two-port is decoupled, i.e., $S_{12} = S_{21} = 0$.

The two-circuit configuration mentioned above, namely a hybrid with the DUT and a resistive termination in one case, and with the DUT, a reactive termination and two isolators in the other, are matched linear passive two-ports which, since all components are cooled, are at a homogeneous temperature. Thus, according to Bosma's theorem, the correlation of the noise waves must be zero.

However, the theorem no longer holds if the two-port is not at a uniform temperature. Consequently, Sutherland and van der Ziel obtain a correlation for a hybrid with a cold DUT and reactive termination, but uncooled isolators.

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H. J. Siweris and B. Schiek are with the Institut fur Hoch- und Höchstfrequenztechnik, Ruhr-Universität Bochum, 4630 Bochum 1, Federal Republic of Germany.

K. M. Lüdeke is with Philips Forschungslaboratorium Hamburg, 2000 Hamburg 54, Federal Republic of Germany.

[1] A. D. Sutherland and A. van der Ziel, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, no. 5, pp. 715–718, May 1982.

Reply[2] *by A. D. Sutherland and A. van der Ziel*[3]

We wish to thank H. J. Siweris, B. Schiek, and K. M. Lüdeke for calling our attention to Bosma's work, of which we were unaware. On checking that reference we found it to be a lengthy tome indeed, consisting of some 190 pages. It is packed with matrix equations which do not readily reveal their meaning upon first reading. In fact, it appears to be unabridged version of Bosma's Ph.D. dissertation. Tucked away as it is within those 190 pages, it is not surprising that the theorem cited by Siweris *et al.* has "not received the appreciation it deserves."

Although the treatment in our paper focused exclusively upon the noise problems introduced by utilizing a hybrid junction as a power divider, and therefore is not as general as the theorem cited due to Bosma, there are nonetheless advantages to the approach followed by us, i.e., a) the use of signal flow graphs, as used by us, avoids the need for matrix algebra, b) the *sources* of the noise emanating from the hybrid are readily identified, and c) there is no need to assume a homogeneous temperature for those noise sources, as does the theorem cited.

References

[1] H. Bosma, "On the theory of linear noisy systems," *Philips Res. Rep. Suppl.*, 1967, no. 10.

[2] Manuscript received December 2, 1982.

[3] The authors are with the University of Minnesota, Department of Electrical Engineering, Minneapolis, MN 55455

Comments on "The Dynamical Behavior of a Single-Mode Optical Fiber Strain Gage"

PATRICIO A. A. LAURA AND JOSÉ L. POMBO

I. Introduction

In the above paper,[1] Martinelli [1] presents a very interesting and useful comparison between the dynamical response of a single-mode fiber optic and resistive strain gages in the frequency range 25–250 Hz. As shown by the author, the frequency spectrum of the phase change signal and the resistive strain gage signal are in very good agreement.

It is the purpose of the present letter to discuss two points which, in the opinion of the writers, need further clarification: a) the validity of the mechanical analysis with special reference to the single-mode approximation; and b) the effect of an axial force which may be present if the mechanical boundary conditions restrain the axial displacements of the structural element.

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The authors are with the Institute of Applied Mechanics, Puerto Belgrano Naval Base, 8111 Argentina.

[1] M. Martinelli, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, no. 4, pp. 512–516, Apr. 1982.

II. The Single-Mode Approximation

Transverse vibrations of the simply supported beam shown in Fig. 1(a) are described by the well-known Lagrange-Sophie Germaine's mathematical model[2]

$$EI\frac{\partial^4 s}{\partial x^4}+\rho A_0\frac{\partial^2 s}{\partial t^2}=F_0\cos 2\pi\nu t\cdot\delta(x-a_1) \tag{1}$$

where E is Young's modulus, I is the moment of inertia, ρ is the mass density, A_0 is the cross-sectional area, and $\delta(x-a_1)$ is the Dirac's delta function ($x=a_1$).

The governing boundary conditions are

$$s(0,t)=s(L,t)=0 \tag{2a}$$

$$\frac{\partial^2 s}{\partial x^2}(0,t)=\frac{\partial^2 s}{\partial x^2}(L,t)=0. \tag{2b}$$

The forced vibrations situation is easily solved expanding Dirac's delta function in terms of the normal modes of the structure ($\sin n\pi x/L$), resulting then in

$$\delta(x-a)=\frac{2}{L}\sum_{n=1}^{\infty}\sin\frac{n\pi a_1}{L}\sin\frac{n\pi x}{L}. \tag{3}$$

Since

$$s(x,t)=\cos 2\pi\nu t\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L} \tag{4}$$

one obtains, substituting (3) and (4) in (1)

$$b_n=\frac{2}{L}\frac{F_0}{\rho A_0}\frac{\sin\frac{n\pi a_1}{L}}{\omega_n^2-\omega^2} \tag{5}$$

where

$\omega=2\pi\nu$ (forcing circular frequency), and

$\omega_n=\sqrt{\frac{EI}{\rho A_0}}\left(\frac{n\pi}{L}\right)^2$ (natural frequencies of the system).

Accordingly

$$s(x,t)=\frac{2}{L}\frac{F_0}{\rho A_0}\cos\omega t\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi a_1}{L}}{\omega_n^2-\omega^2}\sin\frac{n\pi x}{L}. \tag{6}$$

The stress resultants (commonly used in engineering practice) are now as follows.

Bending Moment

$$M(x,t)=-EI\frac{\partial^2 s}{\partial x^2}$$

$$=2\frac{EI}{L}\frac{F_0}{\rho A_0}\cos\omega t\sum_{n=1}^{\infty}\left(\frac{n\pi}{L}\right)^2\frac{\sin\frac{n\pi a_1}{L}}{\omega_n^2-\omega^2}\sin\frac{n\pi x}{L}. \tag{7a}$$

Shear Force

$$Q(x,t)=-EI\frac{\partial^3 s}{\partial x^3}$$

$$=2\frac{ET}{L}\frac{F_0}{\rho A_0}\cos\omega t\sum_{n=1}^{\infty}\left(\frac{n\pi}{L}\right)^3\frac{\sin\frac{n\pi a_1}{L}}{\omega_n^2-\omega^2}\cos\frac{n\pi x}{L}. \tag{7b}$$

[2]In general, the notation used in [1] is followed.

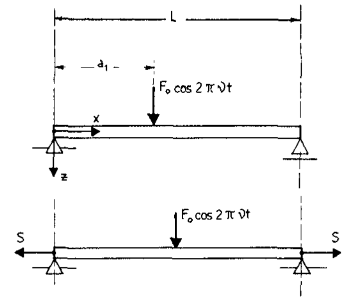

Fig. 1. Vibrating structural element under study: (a) no axial force; (b) axial force present.

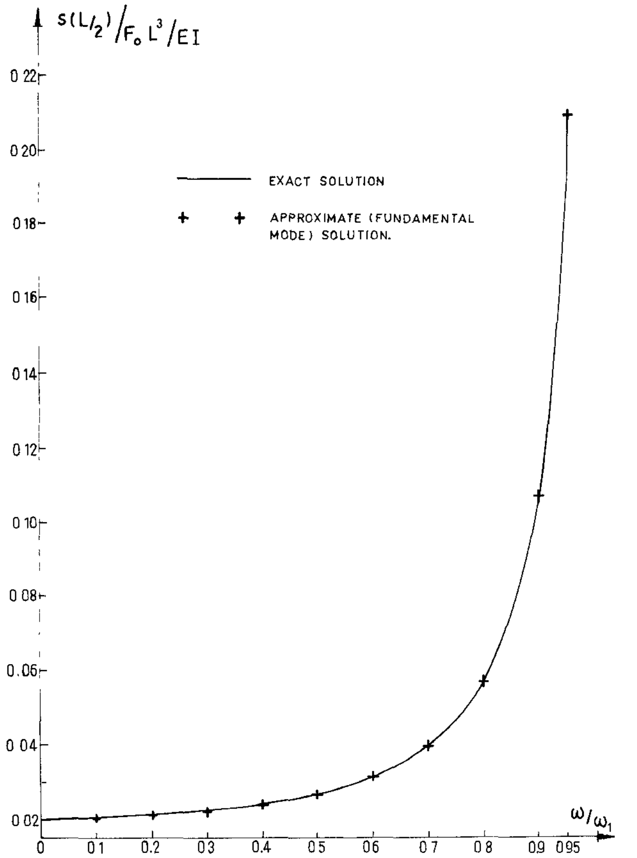

Fig. 2. Amplitude of the dynamic displacement at the center of the beam ($a_1=L/2$).

The strain ϵ_{xx} is simply

$$\epsilon_{xx}(x,t)=\frac{M(x,t)z}{EI} \tag{8a}$$

and the maximum positive strain is attained for $z=a$ ($a=$ semithickness of the beam)

$$\epsilon_{xx}(x,t)=\frac{M(x,t)a}{EI}. \tag{8b}$$

Consequently, the amplitude of the maximum dynamic strain is given by

$$|\epsilon_{xx}(x,t)|=2\frac{a}{L}\frac{F_0}{\rho A_0}\sum_{n=1}^{\infty}\left(\frac{n\pi}{L}\right)^2\frac{\sin\frac{n\pi a_1}{L}}{\omega_n^2-\omega^2}\sin\frac{n\pi x}{L}. \tag{9}$$

Obviously, (9) expresses the fact that the strain is directly proportional to the bending moment at a given cross section.

It should be clear at this point that the displacement amplitudes can be determined with very good accuracy using the first term of (6) for the entire range of frequency values considered by Martinelli [1]. This agrees also with the statement of Timoshenko's classic textbook [2] and is clearly illustrated in Fig. 2 ($a_1=L/2$).

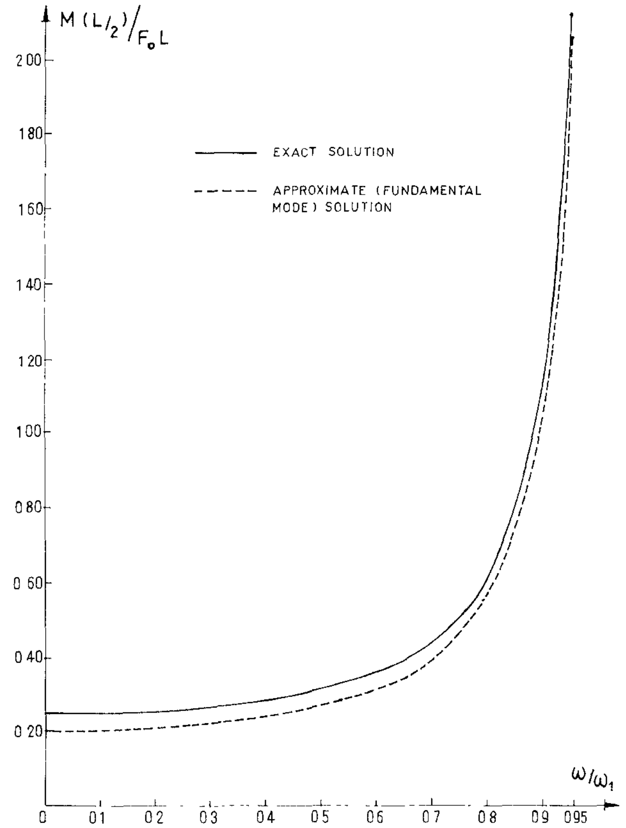

Fig. 3. Amplitude of the dynamic bending moment at the center of the beam ($a_1 = L/2$).

TABLE I
COMPARISON OF BENDING MOMENT AMPLITUDES $M/F_0 \cdot L$ AT THE MIDSPAN OF A VIBRATING BEAM SUBJECTED TO A FORCED EXCITATION AT $X = L/2$

$\frac{\omega}{\omega_1}$	Values of $\frac{M}{F_0 L}$		Error
	Approximate Result	Exact	
0	0.203	0.250	19%
0.1	0.205	0.252	18.7%
0.2	0.211	0.258	18.1%
0.3	0.223	0.270	17.5%
0.4	0.241	0.289	16.5%
0.5	0.270	0.318	15.0%
0.6	0.317	0.364	13.0%
0.7	0.397	0.445	10.7%
0.8	0.562	0.610	8.2%
0.9	1.066	1.114	4.3%
0.95	2.078	2.126	2.2%

On the other hand, it is not permissible to determine strains (or bending moments) using only the fundamental mode shape, as has been done by Martinelli [1], since this generates considerable error as shown in Table I (see also Fig. 3). This fact constitutes a basic theorem of the theory of Fourier series.

Admittedly, the expressions for the phase modulation induced in the laser beam emerging from the fiber cemented on the vibrating bar, and the fiber-optic microstrain obtained by Martinelli [1], may not show considerable numerical difference from a practical viewpoint when the appropriate series expression is used; but the conceptual requirement is very important in the opinion of the writers.

III. Effect of an In-Plane Axial Force

If the boundary constraints restrain the motion of the beam in the x-direction, an axial force S is generated (Fig. 1(b)). In the case of a transversely vibrating beam, this force S will be, in general, a function of time. Equation (1) must now be changed to include the effect of the axial force and now reads

$$EI\frac{\partial^4 s}{\partial x^4} + \rho A_0 \frac{\partial^2 s}{\partial t^2} - S\frac{\partial^2 s}{\partial x^2} = F_0 \cos 2\pi\nu t \cdot \delta(x - a_1). \quad (10)$$

It is easy to show that if $S > 0$ (tensile force), the natural frequencies increase. In essence, the beam behaves as a stiffer structural element.

IV. Conclusions

Distinguished vibrations experts like Leissa [3] agree on the fact that the determination of dynamic stresses and strains in vibrating structural elements is, in general, a difficult task, especially in cases where the exact normal modes are not known. Several studies have been performed at the Institute of Applied Mechanics in recent years [4]–[6] in this area.

Reply[3] by Mario Martinelli[4]

The authors have drawn attention to the mechanical analysis involved in my paper with particular reference to the dynamical strain state of the steel bar, and consequently of the fiber. Their letter reports interesting results, and I think their discussion improves the theoretical treatment of the optical fiber strain sensors.

In my opinion, a realistic prediction of the vibration phenomena requires that the analysis should be completed by taking into account the dynamical damping factor, too.

Since it is difficult to determine this factor with sufficient precision, the choice of a direct comparison [1] between the deformation state of the fiber and that measured using a well-known device (such as the resistive strain gage) seems appropriate to me.

I would like to take this opportunity to make two corrections of typographical errors appearing in [1, p. 514]. On the left column the variable "z" in the expressions (6) and (7) must be read as the lower label of $d(1/n^2)$.

On the right column, row 10, N must be read as μ.

Moreover on page 515, left column, row 10, the phrase "Plots of the rms photodiode signal (full line) and of the rms bridge amplifier (dashed line)" should be read "Plots of the *peak-to-peak* photodiode signal (full line) and of the *peak-to-peak* bridge amplifier (dashed line)", and the vertical scale of Fig. 3 must be intended as labeled in peak-to-peak values.

References

[1] M. Martinelli, "The dynamical behavior of a single-mode optical fiber strain gage," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, no. 4, pp. 512–516, Apr. 1982

[2] S. Timoshenko and D. H. Young, *Vibration Problems in Engineering*. Princeton, NJ: Van Nostrand, 1955, pp. 297–341.

[3] A. W. Leissa, *Vibration of Plates*. NASA SP 160, 1969.

[4] P. A. A. Laura and R. Duran, "A note on forced vibrations of a clamped rectangular plate," *J. Sound and Vibration*, vol. 42, no. 1, pp. 129–135, 1975.

[5] E. A. Susemihl and P. A. A. Laura, "Forced vibrations of thin, elastic, rectangular plates with edges elastically restrained against rotation," *J. Ship Research*, vol. 21, no. 1, pp. 24–29, 1977.

[6] P A. A. Laura, E. A. Susemihl, J. L. Pombo, L. E. Luisoni, and R. Gelos, "On the dynamic behavior of structural elements carrying elastically mounted concentrated masses," *Appl. Acoust.*, vol. 10, pp. 121–145, 1977.

[3] Manuscript received December 2, 1982.
[4] The author is with CISE S.P.A., P.O.B. 12081, 20134, Milano, Italy.